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Full Length Research Paper

Transportation cost in multi-item economic order quantity

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This paper addresses a supplier-retailer logistic system for multi-item as a two-echelon environment. There is a single location in each echelon; the unique supplier at the first echelon has to replenish the retailer's warehouse at the second echelon. We present the model which involves multi stage shipment with a specific number of vehicles and an algorithm is presented for solving the model. Computational results were used to verify the proposed model as well as the efficiency of the algorithm.

Keywords: Logistics management, multi-item, integrated models, transportation cost.

INTRODUCTION

Today's competitive environment speeds up the process of the designing, manufacturing and distributing of products. Simultaneously, severe needs for higher efficiency and lower operational cost are increasing. Such factors are compelling enterprises to continuously look for ways to improve their operations. Companies utilize tools such as optimization models and algorithms, decision support systems and computerized analysis to improve their operational performance and stay competitive. Lately, new approach which is based on integration of decisions of different functions (such as supply process, distribution, inventory management, production planning, facilities location, etc.) into a single optimization model for analyzing of production and distribution operations has been presented. This approach seems to be of meaningful relevance to companies which have adopted it. Blumenfeld et al. (1987), King and Love (1980) and Martin et al. (1993) present three cases which take advantages from applying this integrated analysis to their operations and developed decision support tools.

Along with this flow, this paper investigated the integration of production, inventory and transportation arising in a supplier–retailer logistic system. When products are delivered from the supplier to the consumer, transportation costs are incurred. These costs have been calculated with the production cost or with the ordering cost in the traditional economic order quantity (EOQ) model. But in a practical logistic system, the transportation cost of a vehicle includes both of the fixed cost and the variable cost. The fixed cost refers to essential expenses, as parking fare and wages to driver should have constant sum in each period. Besides the fact that the variable cost is the cost directly associated with the distance traveled, it depends on consumption of oil. With respect to real condition, supposing the transportation cost is proportionate to the quantity delivered or considered as a constant sum is not reasonable. That is, the optimal ordering quantity y^* obtained based on the general EOQ formula may be partly loaded by the vehicles and the logistic systems cost may not be the lowest.

In reviewing the literature of Production-Inventory-Distribution-Inventory models, Zhao et al. (2004) proposed a model which has two-echelon systems and considered optimizing both the inventory cost in the second echelon and the transportation cost in the first echelon. Their model involved the fixed cost of the vehicles, likewise the variable cost for single item. Mak and Wong. (1995) utilize genetic algorithm for solving the

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inventory-production-distribution problem. They involved three echelons in their model which consists of several suppliers, one manufacturing plant and several retailers. The goal of their investigation was to simultaneously optimize stock levels, production quantities and transportation quantities and also minimized total system costs which composed of inventory holding, shortage, manufacturing and transportation costs. Yano et al. (1989) proposed a methodology that simultaneously specify safety stock level at the location in the second echelon (customer), number of vehicles required for regular delivery and time between shipments in a way to minimize overall operational costs.

In the context of Production-Inventory-Distribution-Inventory models, Blumenfeld et al. (1985) engage in analyzing the existing trade-offs between transportation, inventory holding and production set-up costs in the network. They calculated shipment sizes that trade-off the costs of shipment in the cases of direct shipping between nodes in the echelons, shipping through a consolidation terminal and a combination of both. The issue of combined inventory and vehicle routing problems, which addresses the coordination of inventory and transportation management has been a popular topic with the researchers. Federgruen et al. (1995) present a comprehensive review in this domain. These problems are Called Inventory Routing Problems (IRP) and are widely used in VMI partnerships. These issues can also be categorized as an example of channel coordination problem (CCP) which has been performed by both marketing and supply chain researchers. Huang and Lin (2010) present an integrated model that schedules multi-item replenishment with uncertain demand to determine delivery routes and truck loads. They utilized Ant colony algorithm for solving the model. Liu and Chen (2011) proposed a mathematical model for the inventory routing and pricing problem (IRPP). They compared the result of the proposed heuristic method with that of two other methods in solving the model. In keeping with this trend, Kutanoglu and Lohiya (2008) proposed an optimization model for an integrated inventory and transportation problem in a single-echelon, multi-facility service parts logistics system with time based service level constraints. They conclude that crucial advantages can be gained from transportation mode and inventory integration.

Mendoza and Ventura (2008) developed a traditional economic order quantity model with two modes of transportation, truckload (TL) and less than truckload (LTL) carriers. They used an exact algorithm to calculate optimal policies for single-stage models over an infinite planning horizon. Bard and Nananukul (2010) addressed a previously developed mixed-integer programming (MIP) model which minimizes production, inventory, and delivery costs across the various stages of the system. Their model consists of a single production facility, a set of customers with time varying demand, a finite planning horizon, and a fleet of homogeneous vehicles. They used branch-and-price framework to solve the underlying MIP.

This paper investigates minimizing the production, inventory and transportation costs of supplier-retailer logistic system which has multi-item case by considering both of the fixed and the variable cost. We consider both of the fixed cost and the variable cost in our model. Meanwhile, as regards the multiple use of the vehicle that can share the fixed cost and may reduce the total cost arising in the logistic system, the permitted working duration of the vehicle as well as the travel time of such vehicle along the trip is also taken into account.

Subsequently, a breakdown of these steps is shown in this study:

Step 1: Presenting a proposal model and some lemmas.
Step 2: Using an appropriate algorithm to find the optimal solution for the model.
Step 3: Submitting few examples.
Step 4: Conclusion.

THE MODEL

The assumptions of our model for supplier-retailer problem are as follow:

1. Demand is stagnant during the horizon of planning.
2. Shortage not allowed (the replenishment should be completed before product shortage happening).
3. We have a set of analogous vehicles which have bounded capacity for delivery.
4. The third logistic party supplies our vehicles as the delivery required is being finished.

The purpose of this paper is minimizing the whole average costs of the logistic system during the long horizon of planning. Parameters of the model are as follow:

β_i - represent the demand quantity per unit time (a day) for item *i*.

y_i - indicates ordering quantity of item *i* (When $y = \sum_{i=1}^{w} y_i$ is received, the highest inventory occurs and it decreased to zero after *T* time periods).

p - shows the capacity of the vehicle

f - displays the lowest cost of hiring a vehicle in a working day (fixed cost of a vehicle is independent from the duration it will be traveled).

c - depicts the variable transportation cost per trip.

U - shows work duration per day.

t - denotes duration of each trip.

k - is the major setup cost for the family and k_i is the minor setup cost for item *i*.

We use m_i to show the integer number of *T* intervals that the replenishment quantity of item *i* will lost.

Carrying charge is depicted by *r* and s_i is the production cost of item *i*. Now we modeled the problem as follow (P_1):

Minimize

$$TCU_0(T) = \frac{1}{T}(k + \sum_{i=1}^{w} \frac{k_i}{m_i}) + \frac{1}{T}\sum_{i=1}^{w} s_i \beta_i T + \frac{1}{T} nc + \frac{1}{T} mf + \sum_{i=1}^{w} r s_i \frac{m_i \beta_i}{2} T \quad (1)$$

Subject to:

$$(n-1)\frac{p}{\sum_{i=1}^{w} m_i \beta_i} < T \le n \frac{p}{\sum_{i=1}^{w} m_i \beta_i} \quad (2)$$

$$d \le U/t \quad (3)$$

$$md \ge n \quad (4)$$

m,n,d are integers (5)

In the above criterion, $TCU_0(T)$ is the total cost per unit time related to the logistic system, size of ordering for item *i* is shown by y_i, number of delivering vehicle for *y* is *m* and the total trips of this vehicle is *n*.

The number of trips finished by the vehicles for delivering quantity *y* is determined by constraint (2). Since *d* in constraints (3) and (4) represents the maximum trips each vehicle is able to complete in a working day, we can regard it as a predetermined parameter in the following paragraphs and sections. Let *m = g(n)*, in the following we further express Model P_1 as Model P_2:

P_2 Model:

Minimize *TCU(T)* =

$$\frac{1}{T}(k + \sum_{i=1}^{w} \frac{k_i}{m_i}) + \frac{1}{T}\sum_{i=1}^{w} s_i \beta_i T + \frac{1}{T}(nc + fg(n)) + \sum_{i=1}^{w} r s_i \frac{m_i \beta_i}{2} T \quad (6)$$

Subject to Equations (2) and (5) and the following constraint (7):

$$g(n) = m, \left\lceil \frac{n}{d} \right\rceil \quad (7)$$

TCU(T) is not a continuous function, it cannot be differentiated during the whole interval. However, it is observed that when *n* is fixed, the value of *g(n)* is also a constant. Denote *TCU(T)* with a given *n* as $TCU_n(T)$, that is:

$$TCU_n(T) = \frac{1}{T}(k + \sum_{i=1}^{w} \frac{k_i}{m_i}) + \frac{1}{T}\sum_{i=1}^{w} s_i \beta_i T + \frac{1}{T}(nc + fg(n)) + \sum_{i=1}^{w} r s_i \frac{m_i \beta_i}{2} T \quad (8)$$

It is obvious that when *y*>0, function $TCU_n(T)$ is continuous. To derive formulation (8), let:

$$\frac{dTCU_n(T)}{dT} = 0$$

then:

$$T_n^* = \sqrt{\frac{2[k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n)]}{r\sum_{i=1}^{w} s_i m_i \beta_i}} \quad (9)$$

and

$$TCU_n(T_n^*) = \sqrt{2r(\sum_{i=1}^{w} s_i m_i \beta_i)(k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))} + (\sum_{i=1}^{w} s_i \beta_i) \quad (10)$$

Since variables *n* and *g(n)* are taken as the constant in function $TCU_n(T)$, Model P_2 can be expressed as the following formulation:

$$\min_{n \in N} \begin{cases} \min TCU_n(T) \\ st(2) \end{cases}$$

Based on the above analysis, the following conclusions can be derived:

Conclusion 1. The function $TCU_n(T)$ is convex and there exists a unique lowest solution at the point $T_n = T_n^*$, where T_n^* can be given by formulation (9).

Conclusion 2. The optimal solution of Model P_2 can be obtained by the following steps:

1. For different positive integer *n*, the lowest solution $TCU_n(T)$ which satisfies constraint (2) is searched for, and this value is denoted as *f (n)*.
2. The optimal solution of Model P_2 can be obtained by

comparing all of *f(n)*, $n \in N$

We wish to select the m_i's to minimize $TCU_n(T_n^*)$. From an inspection of Equation 10, this is achieved by selecting the m_i's to minimize:

$$F(m_i\text{'s}) = (k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))(\sum_{i=1}^{w} s_i m_i \beta_i) \quad (11)$$

The minimization of Equation 11 is no simple matter because of two facts: (1) the m_i's interact (that is, the effects of one m_i value depend on the values of the other m_i's) and (2) the m_i's must be integers (Schweitzer and Silver, 1983).

If we choose to ignore the integer constraints on the m_i's and set partial derivatives of $F(m_i\text{'s})$ equal to zero (necessary conditions for a minimum), then:

$$\frac{\partial F(m_i\text{'s})}{\partial m_j} = \frac{-k_j}{m_j^2} \sum_{i=1}^{w} s_i m_i \beta_i + s_j \beta_j (k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n)) = 0$$

Or:

$$m_j^2 = \frac{k_j \sum_{i=1}^{w} s_i m_i \beta_i}{s_j \beta_j (k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))} \quad (12)$$

For $j \neq k$, we have:

$$m_k^2 = \frac{k_k \sum_{i=1}^{w} s_i m_i \beta_i}{s_k \beta_k (k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))}$$

Dividing gives:

$$\frac{m_j^2}{m_k^2} = \frac{k_j}{s_j \beta_j} \frac{s_k \beta_k}{k_k}$$

Or:

$$\frac{m_j}{m_k} = \sqrt{\frac{k_j}{s_j \beta_j} \frac{s_k \beta_k}{k_k}} \quad j \neq k$$

We can see that if:

$$\frac{k_j}{s_j \beta_j} < \frac{s_k \beta_k}{k_k}$$

Then the continuous solution of m_j is less than the continuous solution of m_k. Therefore, the item *i* having the smallest value of $\frac{k_i}{s_i \beta_i}$ should have the lowest value of m_i - namely, item 1. It is reasonable to assume that this will hold even when the m_j's are restricted to being integers.

If the items are numbered such that item 1 has the smallest value of $\frac{k_i}{s_i \beta_i}$, then

$m_1 = 1$.

And, from Equation 12:

$$m_j = \sqrt{\frac{k_j}{s_j \beta_j}} \sqrt{\frac{\sum_{i=1}^{w} s_i m_i \beta_i}{(k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))}} \quad (13)$$

Suppose that there is a solution to these equations, it results in:

$$\sqrt{\frac{\sum_{i=1}^{w} s_i m_i \beta_i}{(k + \sum_{i=1}^{w} \frac{k_i}{m_i} + nc + fg(n))}} = A \quad (14)$$

Then, from Equation 13, we have:

$$m_j = A \sqrt{\frac{k_j}{s_j \beta_j}} \quad (15)$$

Therefore:

$$\sum_{i=1}^{w} s_i m_i \beta_i = s_1 \beta_1 + \sum_{i=2}^{w} s_i A \sqrt{\frac{k_i}{s_i \beta_i}} \beta_i = s_1 \beta_1 + A \sum_{i=2}^{w} \sqrt{k_i s_i \beta_i} \quad (16)$$

Similarly,

$$\sum_{i=1}^{w}\frac{k_i}{m_i}=k_1+\frac{1}{A}\sum_{i=2}^{w}\sqrt{k_i s_i\beta_i} \qquad (17)$$

Substituting Equations 16 and 17 back into the left-hand side of Equation 14 and squaring it, we obtain:

$$\frac{s_1\beta_1+A\sum_{i=2}^{w}\sqrt{k_i s_i\beta_i}}{(k+k_1+\frac{1}{A}\sum_{i=2}^{w}\sqrt{k_i s_i\beta_i}+nc+fg(n)}=A^2$$

Cross-multiplication gives:

$$A=\sqrt{\frac{s_1\beta_1}{k+k_1+nc+fg(n)}}$$

Substitution of this expression back into Equation 15 gives:

$$m_j=\sqrt{\frac{s_1\beta_1}{k+k_1+nc+fg(n)}\frac{k_j}{s_j\beta_j}} \quad j=2,3,...,w \qquad (18)$$

A suggested iterative solution procedure is to initially set $m_1=m_2=...=m_w=1$; then solve for a corresponding $TCU_n(T_n^*)$ value in Equation 10, and then use the found *n* value in Equation 18 to find a new m_j's, and so forth.

Because of the convex nature of the functions involved, convergence to the true simultaneous solution pair (m_j's and $TCU_n(T_n^*)$) is ensured. It is time consuming and impractical to compare *f(n)* for each $n\in N$, however, based on the following theorems, we can limit the number of *n* that is needed to be considered and the optimal solution of Model P_2 can be found within limited steps.

Theorem 1. For any function $TCU_n(T)$, if T_n^* gained by formulation (9) also satisfies constraint (2), then *f(n)*=$TCU_n(T_n^*)$ where the meaning of *f(n)* is the same as that defined in conclusion 2; otherwise, *f(n)*=min{$TCU_n(T_1)$, $TCU_n(T_2)$}, where:

$$T_1=\frac{(n-1)p+1}{\sum_{i=1}^{w}m_i\beta_i},T_2=n\frac{p}{\sum_{i=1}^{w}m_i\beta_i}.$$

Proof. According to conclusion 1, we know that $TCU_n(T)$ is convex and there exists a unique optimal solution at $T_n=T_n^*$. If T_n^* is within the interval given by constraint (2), clearly *f(n)*=$TCU_n(T_n^*)$. On the other hand, if T_n^* is not within the interval given by constraint (2), since the function $TCU_n(T)$ is either increased or decreased within the given interval, we can find the lowest value of $TCU_n(T)$ by comparing the value of the two side nodes of the interval.

Theorem 2. If $f(n_j)$ satisfies either of the following two conditions, then for all $n_i>n_j$, the lowest value of $TCU_{n_i}(T)$ that is, $f(n_i)$, cannot be lower than $f(n_j)$:

1. $f(n_j)=TCU_{n_j}(T_{n_j}^*)$;
2. $f(n_j)\le TCU_{n_k}(T_{n_k}^*)$, where $n_k=n_j+1$.

Proof. 1. As *g(n)* is a non-decreased function of *n*, it can be seen from formulations (by consideration (9) and (10) that, with the increment of *n*, T_n^* and $TCU_n(T_n^*)$ become larger. So, for all $n_i>n_j$, $TCU_{n_j}(T_{n_i}^*)>TCU_{n_j}(T_{n_j}^*)$ then $f(n_i)\ge TCU_{n_i}(T_{n_i}^*)>TCU_{n_j}(T_{n_j}^*)=f(n_j)$.
2. It can be discovered from the given condition that $f(n_j)\le TCU_{n_k}(T_{n_k}^*)\le f(n_k)$. Since $TCU_{n_k}(T_{n_k}^*)$ is a non-decreased function of n_k, it can be derived that for all $n_i>n_k$, $f(n_j)\le TCU_{n_k}(T_{n_k}^*)<TCU_{n_i}(T_{n_i}^*)\le f(n_i)$.

Corollary. If $TCU_{\min}(n_j)\le TCU_{n_k}(T_{n_k}^*)$, where $TCU_{\min}(n_j)=\min\{f(n_r)\mid$ for all $n_r\le n_j\}$, then for all $n_i>n_k, TCU_{\min}(n_j)\le TCU_{n_k}(T_{n_k}^*)<TCU_{n_i}(T_{n_i}^*)\le f(n_i)$.

Proof. It can be deduced from the above theorems.

THE ALGORITHM

Now the appropriate algorithm is presented as follows:

Step 1: Set $m_1=m_2=...=m_w=1$

Step 2: Let $n = \min\{1, \frac{\sum_{i=1}^{w} m_i \beta_i}{p}\}$.

Step 3: Calculate $T_n^* = \sqrt{\frac{2[k + \sum_{i=1}^{w} \frac{k_i}{m_i} + cn + fg(n)]}{r \sum_{i=1}^{w} s_i m_i \beta_i}}$.

Step 4: If T_n^* satisfies constraint $\frac{(n-1)p}{\sum_{i=1}^{w} m_i \beta_i} < T \le n \frac{p}{\sum_{i=1}^{w} m_i \beta_i}$, go to Step 5, else go to Step 7

Step 5: Record $f(n) = TCU_n(T_n^*) = \sqrt{2r(\sum_{i=1}^{w} s_i m_i \beta_i)(k + \sum_{i=1}^{w} \frac{k_i}{m_i} + cn + fg(n))} + (\sum_{i=1}^{w} s_i \beta_i)$, $TCU_{\min} = \min\{TCU_{\min}, f(n)\}$, (the initial value of $TCU_{\min}$ is set to infinite).

Step 6: Use found *n* in Equation 18 and obtain m_j. If there is no changes in m_j values then stop, else go to Step 2.

Step 7: Calculate $T_1 = \frac{(n-1)p+1}{\sum_{i=1}^{w} m_i \beta_i}, T_2 = n \frac{p}{\sum_{i=1}^{w} m_i \beta_i}$.

$f(n) = \min\{TCU_n(T_1), TCU_n(T_2)\}$,
$TCU_{\min} = \min\{TCU_{\min}, f(n)\}$, (the initial value of $TCU_{\min}$ is set to infinite).

Step 8: Calculate $TCU_{n_k}(T_{n_k}^*)$ for $n_k = n+1$.

Step 9: If $TCU_{\min} \le TCU_{n_k}(T_{n_k}^*)$ then go to Step 10 else *n*=*n*+1 and go to Step 3.

Step 10: Use found *n* in Equation 18 and obtain m_j. If there is no changes in m_j values then stop, else go to Step 2.

THE EXAMPLES

For verifying the efficiency of the given model as well as the algorithm, we provide three examples. First example is based on this assumption that the transportation cost is proportional to the quantities delivered and no traveling duration constraint is considered. In the second example, we use travel distance of the vehicles as a base for calculating the transportation cost and the fixed cost is not considered. In the third example, we consider the transportation costs composed of the fixed cost, only the fixed cost which is a fixed sum whenever a vehicle is employed, but also the variable cost which is calculated based on the travel distance of the vehicle. Furthermore we consider the permitted working duration as well as the travel time of any vehicle along the trip in the last two examples.

The parameters in these examples are the same as the prior section. In order to simplify our process, the units of the parameters in the examples are not considered, with respect to the fact that the computational results as well as the conclusions cannot be affected by such simplification.

In the first example, we track procedure (11) to find the best set of m_i's and y_i^*'s because the transportation cost is assumed to be proportional to the quantities delivered and no traveling duration constraint is considered.

Step 1. Evaluate $\frac{k_i}{s_i \beta_i}$ for all items such that $\frac{k_i}{s_i \beta_i}$ is smallest for item *i*. Set $m_i = 1$

Step 2. For $j \ne i$ Evaluate $m_j = \sqrt{\frac{k_j}{s_j \beta_j} \frac{s_i \beta_i}{k_i + k}}$ rounded to the nearest integer greater than zero.

Step 3. Evaluate T^* using the $T^* = \sqrt{\frac{2(k + \sum_{i=1}^{w} \frac{k_i}{m_i})}{r \sum_{i=1}^{w} s_i m_i \beta_i}}$

Step 4. Determine $y_i^* = m_i \beta_i T^*$

Example 1

It is assumed that

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k = 55, k_1 = 15, k_2 = 10, k_3 = 20, r = 0.077,$ $s_1 = 0.25, s_2 = 0.20, s_3 = 0.30$. Furthermore, the transportation cost is assumed to be proportional to the quantity delivered and the unit transportation cost, defining by *a*, is equal to 0.2. The aim is to determine the optimal value of T^* and y_i^* (*i*=1,2,3) by considering minimizing the total average cost of the logistic system, where T^* is the ordering cycle, y_i^* is the economic order quantity for item *i*. Now we specify T^*, y_i^* based on the above procedure:

$$\frac{k_1}{s_1\beta_1} = 2, \frac{k_2}{s_2\beta_2} = 2, \frac{k_3}{s_3\beta_3} = 1.48 \Rightarrow m_3 = 1$$ and

$$m_1 = \sqrt{\frac{k_1}{s_1\beta_1}\frac{s_3\beta_3}{k_3 + k}} = 0.36 \Rightarrow m_1 = 1$$ and

$$m_2 = \sqrt{\frac{k_2}{s_2\beta_2}\frac{s_3\beta_3}{k_3 + k}} = 0.36 \Rightarrow m_2 = 1$$

Thus in each ordering cycle, items 1, 2 and 3 are ordered:

$$T^* = \sqrt{\frac{2(k + \sum_{i=1}^{3}\frac{k_i}{m_i})}{r\sum_{i=1}^{3} s_i m_i \beta_i}} = 10,$$

$$y_1^* = 300, y_2^* = 250, y_3^* = 450$$ and

$$y^* = \sum_{i=1}^{3} y_i^* = 1000$$

$$TCU(T^*) = \sqrt{2r(\sum_{i=1}^{3} s_i m_i \beta_i)(k + \sum_{i=1}^{3}\frac{k_i}{m_i})} + \sum_{i=1}^{3}(s_i + a)\beta_i = 20 + 46 = 66$$

Considering the results in the optimal solution, an order for 1000 unit products consisting of 300 unit item 1, 250 unit item 2, and 450 unit item 3, should be sent. The optimal ordering cycle is 10 days and the total transportation cost in each ordering cycle is 200. According to the results, we present the next example.

Example 2

It is assumed that $\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k = 55, k_1 = 15, k_2 = 10, k_3 = 20, r = 0.077,$

$s_1 = 0.25, s_2 = 0.20, s_3 = 0.30, p = 200, c = 40, f = 0, u = 8, t = 4$

. We want to optimize the value of T^* and y_i^* (*i* =1, 2, 3).

The algorithm described in Section 3 is coded by MATLAB. Table 1 depicts the computational results.

According to the results and based on the stopping criterion in Step 10, the algorithm stops when *n*=8. The optimal solution occurs at *n*=5, and $T^* = 10$, $y_1^* = 300, y_2^* = 250, y_3^* = 450$, $y^* = 1000$, $TCU_{\min}$ = 66.00. Three vehicles are used for delivery.

The results of the above examples are equal because the second example is designed based on the results of the first example; the given parameters in Example 2 ensure that the vehicle used for delivery of y^* in Example 1 is fully loaded. However, in Example 2, when *n* = 5, the value of T_n^* calculated according to formulation (9) is 17.32, whereas based on the algorithm, T^* equals to 10. Therefore, the method which used the traditional economic ordering quantity formula is not proper for the given example.

Results show that for all $n \le 8$, $f(n) > TCU_n(T_n^*)$, and the value of $TCU_n(T_n^*)$ increase along with growth of *n*. The following computation indicates that when *n* = 12, $f(n) = TCU_n(T_n^*) = 74.16 > TCU_{\min}$. The outcomes confirm the algorithm.

Example 3

We have

$\beta_1 = 30, \beta_2 = 25, \beta_3 = 45, k = 55, k_1 = 15, k_2 = 10, k_3 = 20, r = 0.077, s_1 = 0.25,$

$s_2 = 0.20, s_3 = 0.30, p = 200, c = 40, f = 30, u = 8, t = 4$

. The computational results are presented in Table 2.

The results show that based on the criterion in Step 10, the algorithm stops when *n*=9. The optimal solution occurs at *n*=6, and $T^* = 12$, $y_1^* = 360, y_2^* = 300, y_3^* = 540$, $y^* = 1200$, $TCU_{\min}$ = 73.83. Three vehicles are used for the delivery. The computation indicates that when *n* = 16, $f(n) = TCU_n(T_n^*) = 92.06 > TCU_{\min}$.

By considering the fixed cost of the vehicle, the value of T^* and y^* is different from that of the obtained results in Example 2. It shows that when *n* = *5*, one of the used vehicles will only finish one trip so the marginal cost of delivering the unit product by this vehicle will be higher than that of the other vehicle which can complete two trips. Furthermore, the inventory quantities may increase by utilizing the vehicle to the greatest efficiency. So the optimal solution of the problem is the results of the trade-off of the transportation cost and the inventory cost.

CONCLUSION

This paper addressed inventory problem in multi-item supplier-retailer logistic system through integration of production, inventory and transportation. We considered both fixed and variable transportation cost. Also we considered multiple use of the vehicle which can share

Table 1. Computational results of Example 2.

n	T_1	T_2	T_n^*	$TCU_n(T_n^*)$	$TCU_n(T_1^*)$	$TCU_n(T_2^*)$	f(n)	$TCU_{\min}$
1	0.01	2.00	11.83	49.66	14026.01	98.00	98.00	98.00
2	2.01	4.00	13.42	52.83	117.56	75.00	75.00	75.00
3	4.01	6.00	14.83	55.66	84.87	68.66	68.66	68.66
4	6.01	8.00	16.13	58.24	75.27	66.50	66.50	66.50
5	8.01	10.00	17.32	60.64	71.46	66.00	66.00	66.00
6	10.01	12.00	18.44	62.87	69.97	66.33	66.33	66.00
7	12.01	14.00	19.50	64.98	96.65	67.14	67.14	66.00
8	14.01	16.00	20.50	66.98	-	-	-	-

Table 2. Computational results of Example 3.

n	T_1	T_2	T_n^*	$TCU_n(T_n^*)$	$TCU_n(T_1^*)$	$TCU_n(T_2^*)$	f(n)	$TCU_{\min}$
1	0.01	2.00	13.04	52.07	17026.01	113.00	113.00	113.00
2	2.01	4.00	14.49	54.98	132.49	82.50	82.50	82.50
3	4.01	6.00	16.74	59.46	99.83	78.66	78.66	78.66
4	6.01	8.00	17.89	61.77	85.25	74.00	74.00	74.00
5	8.01	10.00	19.75	65.49	82.70	75.00	75.00	74.00
6	10.01	12.00	20.74	67.47	78.96	73.83	73.83	73.83
7	12.01	14.00	22.36	70.71	79.64	75.71	75.71	73.83
8	14.01	16.00	23.24	72.47	78.55	75.75	75.75	73.83
9	16.01	18.00	24.70	75.39	-	-	-	-

the fixed transportation cost. The proposal model which finds the best trading-off of all costs related to the logistic system is presented and the suitable algorithm for such model is shown. Thus, the computational results verify the proposed model as well as the efficiency of the algorithm.

For future research, the following suggestions are made:

1. More than one retailer or more than one supplier can be considered in the model.
2. Through the further investigation of the relationship of each item in the model, more efficient algorithms can be proposed.

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